Ch. 2 Fundamentals

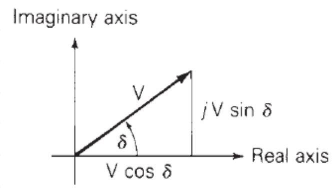
Note Title 2/1/2014

2.1 Phasors:

$v(t) = \mathrm{V}_{\max}\cos(\omega t + \delta)$

$\mathrm{V} = \dfrac{\mathrm{V}_{\max}}{\sqrt{2}}$ (RMS)

$V = \underbrace{\mathrm{V}e^{j\delta}}_{\text{exponential}} = \underbrace{\mathrm{V}\angle\delta}_{\text{polar}} = \underbrace{\mathrm{V}\cos\delta + j\mathrm{V}\sin\delta}_{\text{rectangular}}$



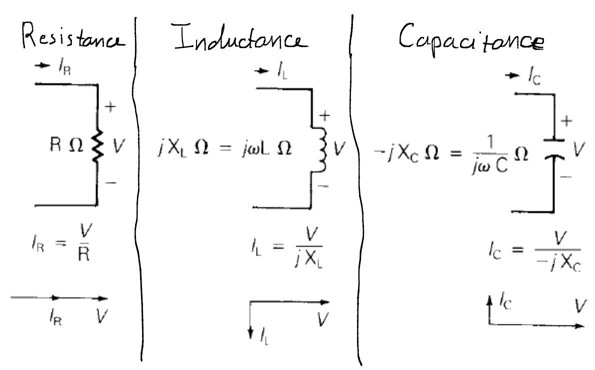
Ex: $v(t) = 169.7\cos(\omega t + 60^\circ)$ volts

$V_{max} = 169.7\,V$, $V = \dfrac{169.7}{\sqrt{2}} = 120\,V$, $\delta = 60^\circ$, $\overbrace{V = 120\angle 60^\circ\,V}^{\text{voltage phasor}}$

$= 60 + j103.92\,V$

FIGURE 2.2

Summary of relationships between phasors V and I for constant R, L, and C elements with sinusoidal-steady-state excitation



$v(t)$ intsantaneous values

(uppercase) V rms values *

V rms phasors

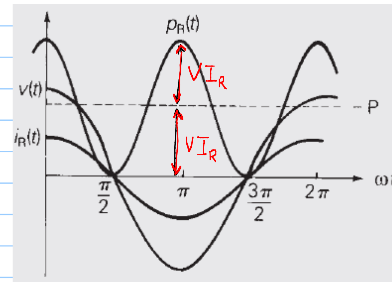
(italic)

2.2 Instantaneous power in single-phase AC circuits:

Power is the rate of change of energy with respect to time.

$v(t) = \mathrm{V_{max}} \cos(\omega t + \delta)$ volts

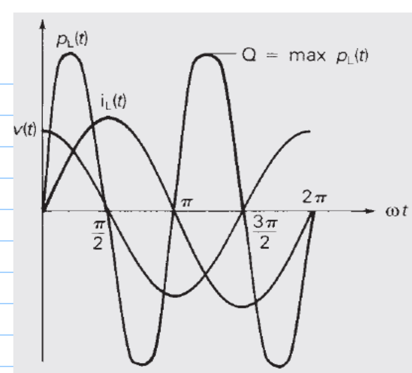
1) Purely Resistive Loads



$i_R(t) = \mathrm{I_{Rmax}} \cos(\omega t + \delta)$ A

$p_R(t) = v(t) i_R(t) = \underbrace{\mathrm{VI_R}}_{\substack{P_R \\ \text{(average value)}}} + \underbrace{\mathrm{VI_R} \cos[2(\omega t + \delta)]}_{\text{(double-frequency term)}}$ W

2) Purely Inductive Loads:



$i_L(t) = \mathrm{I_{Lmax}} \cos(\omega t + \delta - 90°)$ A

$p_L(t) = v(t) i_L(t) = \mathrm{VI_L} \sin[2(\omega t + \delta)]$ W

(0 average value) (double-frequency term only)

3) Purely Capacitive Loads:

$i_C(t) = \mathrm{I_{Cmax}} \cos(\omega t + \delta + 90°)$ A

$p_C(t) = v(t) i_C(t) = -\mathrm{VI_C} \sin[2(\omega t + \delta)]$ W

(0 average value) (double-frequency term only)

4) General RLC Loads:

$$i(t) = \mathrm{I_{max}} \cos(\omega t + \beta) \quad \mathrm{A}$$

$$p(t) = v(t)i(t) = \underbrace{\mathrm{VI_R}\{1 + \cos[2(\omega t + \delta)]\}}_{p_\mathrm{R}(t)} + \underbrace{\mathrm{VI_X} \sin[2(\omega t + \delta)]}_{p_\mathrm{X}(t)}$$

where

$$\mathrm{I} \cos(\delta - \beta) = \mathrm{I_R} \quad \text{and} \quad \mathrm{I} \sin(\delta - \beta) = \mathrm{I_X}$$

$\mathrm{P} = \mathrm{VI_R} = \mathrm{VI} \cos(\delta - \beta) \quad \mathrm{W}$ (Real power),(Active power).

$\cos(\delta - \beta)$ is called power factor (pf).

$(\delta - \beta)$ is called power factor angle.

If $\delta > \beta$, the pf is Lagging and the Load is inductive.

If $\delta < \beta$, the pf is Leading and the load is capacitive.

$\mathrm{Q} = \mathrm{VI_X} = \mathrm{VI} \sin(\delta - \beta) \quad \mathrm{var}$ (Reactive power)

Read Ex. 2.1 and problem 2.13.

2.3 Complex Power:

$V = \mathrm{V}\underline{/\delta}$ & $I = \mathrm{I}\underline{/\beta}$

$$S = VI^* = [\mathrm{V}\underline{/\delta}][\mathrm{I}\underline{/\beta}]^* = \mathrm{VI}\underline{/\delta - \beta}$$

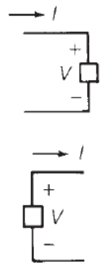
$$= \mathrm{VI} \cos(\delta - \beta) + j\mathrm{VI} \sin(\delta - \beta)$$

$\mathrm{S} = \mathrm{P} + j\mathrm{Q}$ VA (Apparent power)

$P = S \text{ p.f.}$, $Q = S \sin(\delta - \beta)$

FIGURE 2.4

Load and generator conventions



(a) *Load convention.* Current *enters* positive terminal of circuit element. If P is positive, then positive real power is *absorbed*. If Q is positive, then positive reactive power is *absorbed*. If P (Q) is negative, then positive real (reactive) power is *delivered*.

(b) *Generator convention.* Current *leaves* positive terminal of the circuit element. If P is positive, then positive real power is *delivered*. If Q is positive, then positive reactive power is *delivered*. If P (Q) is negative, then positive real (reactive) power is *absorbed*.

resistor: $S_R = VI_R^* = [V\underline{/\delta}]\left[\frac{V}{R}\underline{/-\delta}\right] = \frac{V^2}{R}$ → no reactive power (absorb P)

inductor: $S_L = VI_L^* = [V\underline{/\delta}]\left[\frac{V}{-jX_L}\underline{/-\delta}\right] = +j\frac{V^2}{X_L}$ → no real power (absorb Q)

capacitor: $S_C = VI_C^* = [V\underline{/\delta}]\left[\frac{V}{jX_C}\underline{/-\delta}\right] = -j\frac{V^2}{X_C}$ → no real power (deliver Q)

EXAMPLE 2.2 Real and reactive power, delivered or absorbed

A single-phase voltage source with $V = 100\underline{/130°}$ volts delivers a current $I = 10\underline{/10°}$ A, which leaves the positive terminal of the source. Calculate the source real and reactive power, and state whether the source delivers or absorbs each of these.

SOLUTION Since I leaves the positive terminal of the source, the generator convention is assumed, and the complex power delivered is, from (2.3.1),

$$S = VI^* = [100\underline{/130°}][10\underline{/10°}]^*$$

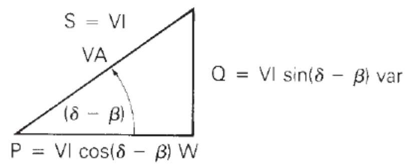
$$S = 1000\underline{/120°} = -500 + j866$$

$$P = \text{Re}[S] = -500 \quad \text{W}$$

$$Q = \text{Im}[S] = +866 \quad \text{var}$$

FIGURE 2.5

Power triangle



$$S = \sqrt{P^2 + Q^2}$$

$$(\delta - \beta) = \tan^{-1}(Q/P)$$

$$Q = P \tan(\delta - \beta)$$

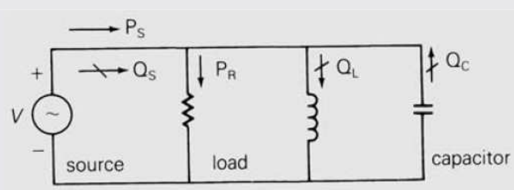
$$\text{p.f.} = \cos(\delta - \beta) = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2}}$$

EXAMPLE 2.3 Power triangle and power factor correction

A single-phase source delivers 100 kW to a load operating at a power factor of 0.8 lagging. Calculate the reactive power to be delivered by a capacitor connected in parallel with the load in order to raise the source power factor to 0.95 lagging. Also draw the power triangle for the source and load. Assume that the source voltage is constant, and neglect the line impedance between the source and load.

FIGURE 2.6

Circuit and power triangle for Example 2.3



$$\theta_L = (\delta - \beta_L) = \cos^{-1}(0.8) = 36.87^\circ$$

$$Q_L = P \tan\theta_L = 100 \tan(36.87^\circ) = 75 \quad \text{kvar}$$

$$S_L = \frac{P}{\cos\theta_L} = 125 \quad \text{kVA}$$

After the capacitor is connected, the power factor angle, reactive power delivered, and apparent power of the source are

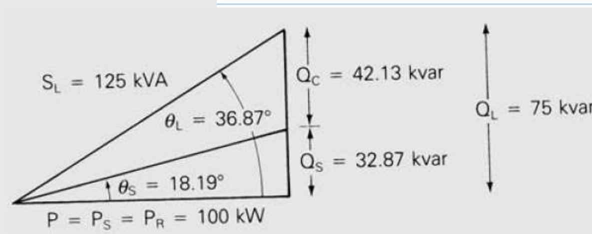
$$\theta_S = (\delta - \beta_S) = \cos^{-1}(0.95) = 18.19^\circ$$

$$Q_S = P \tan\theta_S = 100 \tan(18.19^\circ) = 32.87 \quad \text{kvar}$$

$$S_S = \frac{P}{\cos\theta_S} = \frac{100}{0.95} = 105.3 \quad \text{kVA}$$

The capacitor delivers

$$Q_C = Q_L - Q_S = 75 - 32.87 = 42.13 \quad \text{kvar}$$



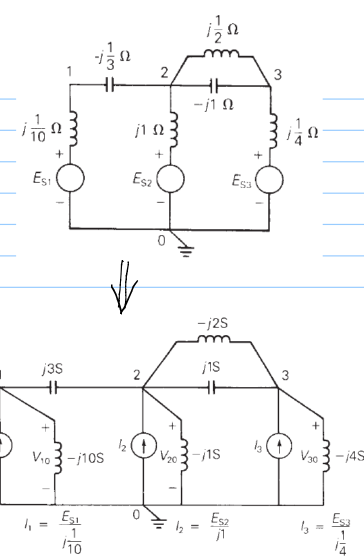
2.4 Network Equations:

Nodal equations are written in the following three steps:

STEP 1 For a circuit with $(N+1)$ nodes (also called buses), select one bus as the reference bus and define the voltages at the remaining buses with respect to the reference bus.

The circuit in Figure 2.8 has four buses—that is, $N+1=4$ or $N=3$. Bus 0 is selected as the reference bus, and bus voltages V_{10}, V_{20}, and V_{30} are then defined with respect to bus 0.

STEP 2 Transform each voltage source in series with an impedance to an equivalent current source in parallel with that impedance. Also, show admittance values instead of impedance values on the circuit diagram. Each current source is equal to the voltage source divided by the source impedance.



STEP 3 Write nodal equations in matrix format as follows:

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & Y_{23} & \cdots & Y_{2N} \\ Y_{31} & Y_{32} & Y_{33} & \cdots & Y_{3N} \\ \vdots & \vdots & \vdots & & \vdots \\ Y_{N1} & Y_{N2} & Y_{N3} & \cdots & Y_{NN} \end{bmatrix}_{N\times N} \begin{bmatrix} V_{10} \\ V_{20} \\ V_{30} \\ \vdots \\ V_{N0} \end{bmatrix}_{N\times 1} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_N \end{bmatrix}_{N\times 1}$$

$$\begin{bmatrix} (j3-j10) & -(j3) & 0 \\ -(j3) & (j3-j1+j1-j2) & -(j1-j2) \\ 0 & -(j1-j2) & (j1-j2-j4) \end{bmatrix} \begin{bmatrix} V_{10} \\ V_{20} \\ V_{30} \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$j \begin{bmatrix} -7 & -3 & 0 \\ -3 & 1 & 1 \\ 0 & 1 & -5 \end{bmatrix} \begin{bmatrix} V_{10} \\ V_{20} \\ V_{30} \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

diagonal elements: Y_{kk} = sum of admittances connected to bus k

self-admittance $(k = 1, 2, \ldots, N)$ (2.4.3)

off-diagonal elements: $Y_{kn} = -$(sum of admittances connected between buses k

mutual-admittance and n) $(k \neq n)$ (2.4.4)

2.5 Balanced Three-phase Circuits:

* Balanced-Y Connections

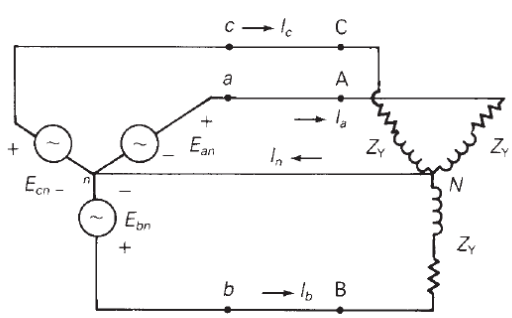
$$E_{an} = 10\underline{/0°}$$

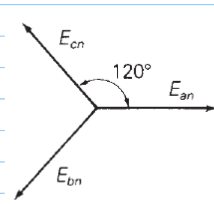
$$E_{bn} = 10\underline{/-120°} = 10\underline{/+240°}$$

$$E_{cn} = 10\underline{/+120°} = 10\underline{/-240°} \quad \text{volts}$$

FIGURE 2.10

Circuit diagram of a three-phase Y-connected source feeding a balanced-Y load





phasor Diagram

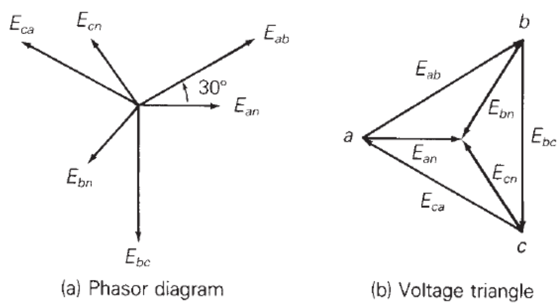
Balanced Voltages:

$$E_{ab} = E_{an} - E_{bn}$$

$$E_{ab} = \sqrt{3}E_{an}\underline{/+30^\circ}$$
$$E_{bc} = \sqrt{3}E_{bn}\underline{/+30^\circ}$$
$$E_{ca} = \sqrt{3}E_{cn}\underline{/+30^\circ}$$

FIGURE 2.12

Positive-sequence line-to-neutral and line-to-line voltages in a balanced three-phase Y-connected system



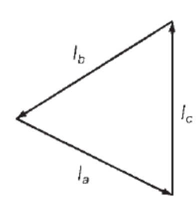
Balanced Currents:

$$I_a = E_{an}/Z_Y$$
$$I_b = E_{bn}/Z_Y$$
$$I_c = E_{cn}/Z_Y$$

$$I_n = I_a + I_b + I_c$$

FIGURE 2.13

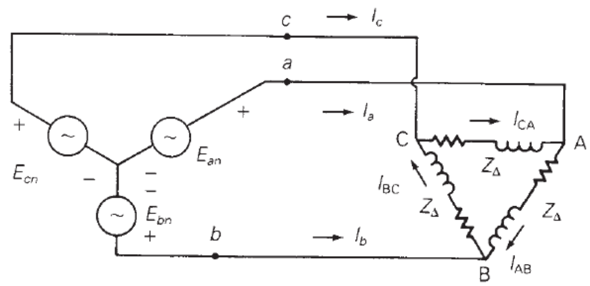
Phasor diagram of line currents in a balanced three-phase system



* Balanced-Δ Loads:

FIGURE 2.14

Circuit diagram of a Y-connected source feeding a balanced-Δ load

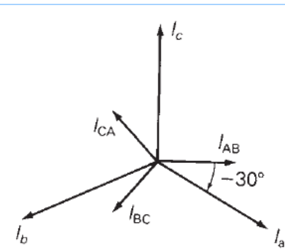


$$I_{AB} = E_{ab}/Z_\Delta$$
$$I_{BC} = E_{bc}/Z_\Delta \quad ,$$
$$I_{CA} = E_{ca}/Z_\Delta$$

$$I_a = \sqrt{3}I_{AB}\underline{/-30^\circ}$$
$$I_b = \sqrt{3}I_{BC}\underline{/-30^\circ}$$
$$I_c = \sqrt{3}I_{CA}\underline{/-30^\circ}$$

FIGURE 2.15

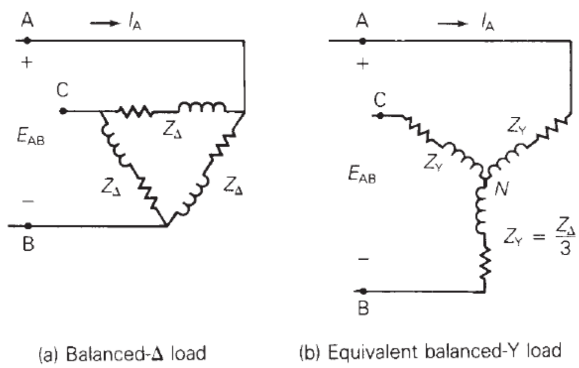
Phasor diagram of line currents and load currents for a balanced-Δ load



Δ-Y Conversion For Balanced Loads:

FIGURE 2.16

Δ–Y conversion for balanced loads

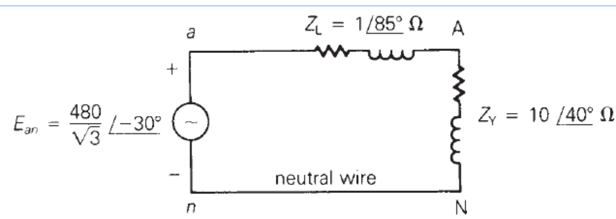


Read Ex 2.4

Equivalent Line-to-Neutral Diagrams

FIGURE 2.18

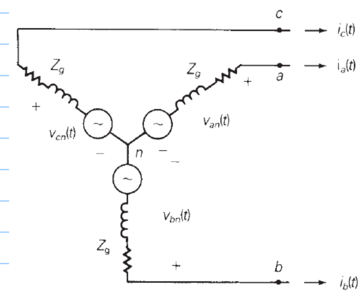
Equivalent line-to-neutral diagram for the circuit of Example 2.4



2.6 Power in Balanced Three-phase Circuits:

* Instantaneous Power (Balanced 3ϕ Generator, Motors and Impedance Loads):

$$v_{an}(t) = \sqrt{2}\mathrm{V_{LN}} \cos(\omega t + \delta) \quad \text{volts}, \quad i_a(t) = \sqrt{2}\mathrm{I_L} \cos(\omega t + \beta) \quad \mathrm{A}$$



$$p_a(t) = v_{an}(t) i_a(t) = \mathrm{V_{LN}I_L} \cos(\delta - \beta) + \mathrm{V_{LN}I_L} \cos(2\omega t + \delta + \beta) \quad \mathrm{W}$$

$$p_b(t) = \mathrm{V_{LN}I_L} \cos(\delta - \beta) + \mathrm{V_{LN}I_L} \cos(2\omega t + \delta + \beta - 240^\circ) \quad \mathrm{W}$$

$$p_c(t) = \mathrm{V_{LN}I_L} \cos(\delta - \beta) + \mathrm{V_{LN}I_L} \cos(2\omega t + \delta + \beta + 240^\circ) \quad \mathrm{W}$$

$$\begin{aligned} p_{3\phi}(t) &= p_a(t) + p_b(t) + p_c(t) \\ &= 3\mathrm{V_{LN}I_L} \cos(\delta - \beta) + \mathrm{V_{LN}I_L}[\cos(2\omega t + \delta + \beta) \\ &\quad + \cos(2\omega t + \delta + \beta - 240^\circ) \\ &\quad + \cos(2\omega t + \delta + \beta + 240^\circ)] \quad \mathrm{W} \quad = \mathrm{P}_{3\phi} = 3\mathrm{V_{LN}I_L} \cos(\delta - \beta) \quad \mathrm{W} = \sqrt{3}\mathrm{V_{LL}I_L} \cos(\delta - \beta) \quad \mathrm{W} \end{aligned}$$

* Complex Power (Balanced 3ϕ Generators, Motor and Balanced-Y Impedance Loads):

$$V_{an} = \mathrm{V_{LN}}\underline{/\delta} \quad \text{volts}$$

$$I_a = \mathrm{I_L}\underline{/\beta} \quad \mathrm{A}$$

$$\begin{aligned} S_a &= V_{an} I_a^* = \mathrm{V_{LN}I_L}\underline{/(\delta - \beta)} \\ &= \mathrm{V_{LN}I_L} \cos(\delta - \beta) + j\mathrm{V_{LN}I_L} \sin(\delta - \beta) = S_b = S_c \end{aligned}$$

Total Complex power $\longrightarrow$

$$\begin{aligned} S_{3\phi} &= S_a + S_b + S_c = 3S_a \\ &= 3\mathrm{V_{LN}I_L}\underline{/(\delta - \beta)} \\ &= 3\mathrm{V_{LN}I_L} \cos(\delta - \beta) + j3\mathrm{V_{LN}I_L} \sin(\delta - \beta) = \overbrace{\sqrt{3}\mathrm{V_{LL}I_L} \cos(\delta - \beta)}^{P_{3\phi}} + j\, \overbrace{\sqrt{3}\mathrm{V_{LL}I_L} \sin(\delta - \beta)}^{Q_{3\phi}} \end{aligned}$$

The total apparent power:

$$\mathrm{S}_{3\phi} = |S_{3\phi}| = 3\mathrm{V_{LN}I_L} = \sqrt{3}\mathrm{V_{LL}I_L} \quad \mathrm{VA}$$

* Complex Power (Balanced-Δ Impedance Load):

$$V_{ab} = \mathrm{V}_{\mathrm{LL}}\underline{/\delta} \quad \text{volts}$$

$$I_{ab} = \mathrm{I}_{\Delta}\underline{/\beta} \quad \mathrm{A}$$

$$S_{ab} = V_{ab}I_{ab}^{*} = \mathrm{V}_{\mathrm{LL}}\mathrm{I}_{\Delta}\underline{/(\delta-\beta)}$$

Total complex power → $S_{3\phi} = S_{ab} + S_{bc} + S_{ca} = 3S_{ab}$

$$= 3\mathrm{V}_{\mathrm{LL}}\mathrm{I}_{\Delta}\underline{/(\delta-\beta)}$$

$$= 3\mathrm{V}_{\mathrm{LL}}\mathrm{I}_{\Delta}\cos(\delta-\beta) + j3\mathrm{V}_{\mathrm{LL}}\mathrm{I}_{\Delta}\sin(\delta-\beta) = \overbrace{\sqrt{3}\mathrm{V}_{\mathrm{LL}}\mathrm{I}_{\mathrm{L}}\cos(\delta-\beta)}^{P_{3\phi}} + j\,\overbrace{\sqrt{3}\mathrm{V}_{\mathrm{LL}}\mathrm{I}_{\mathrm{L}}\sin(\delta-\beta)}^{Q_{3\phi}}$$

Total apparent power:

$$\mathrm{S}_{3\phi} = |S_{3\phi}| = 3\mathrm{V}_{\mathrm{LL}}\mathrm{I}_{\Delta} = \sqrt{3}\mathrm{V}_{\mathrm{LL}}\mathrm{I}_{\mathrm{L}} \quad \mathrm{VA}$$

Read Ex. 2.5